Hale School

Mathematics Specialist

Test 1 --- Term 1 2019

Complex Numbers

/ 46

Name: ______________________________

Instructions:

- **Calculators are NOT allowed**
- **External notes are not allowed**
- **Formula Sheet will be provided**
- **Duration of test: 45 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

All arguments must be given using principal values.

1. **[2, 3 and 3 = 8 marks]**

(a) Find

i) $\text{Re}\left(\dfrac{2+3i}{1-i}\right)$

ii) $\text{Im}\left(2cis\left(\pi/3\right)+3cis\left(\pi/4\right)\right)$

(b) Simplify

$$\frac{\left(\sqrt{2}cis\left(3\pi/4\right)\right)^5}{\left(2cis\left(\pi/6\right)\right)^2}$$ leaving your answer in polar form, $rcis\theta$.

2. [1, 1, 1 and 2 = 5 marks]

In the Argand plane below, a unit circle has been drawn and P is the point corresponding to the complex number z.

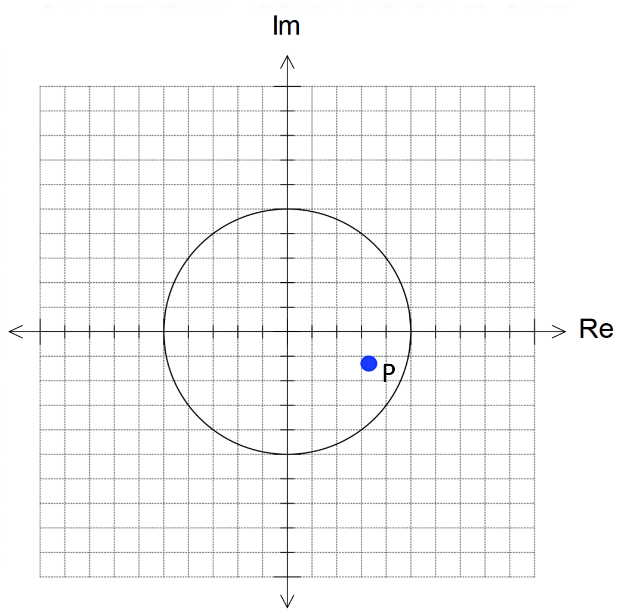
In the diagram, clearly mark the complex numbers corresponding to:

i) z^2

ii) $\frac{1}{z}$

iii) $-2z$

iv) $\bar{z} - z$



3. **[4 marks]**

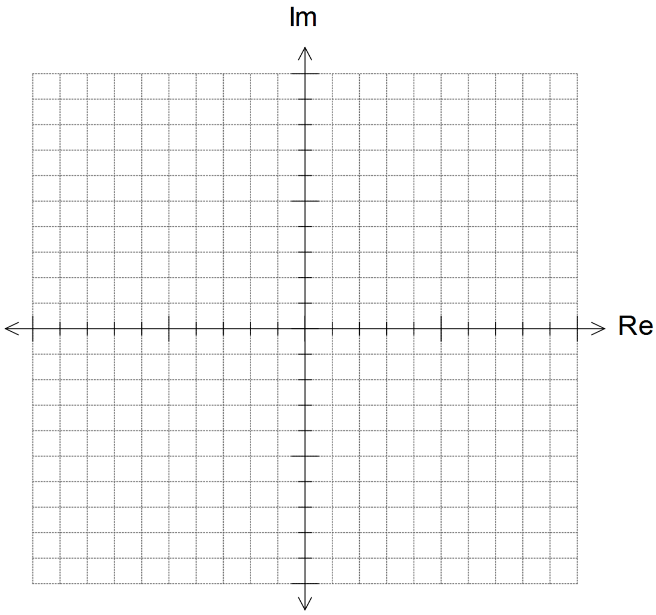
Let $z = a + bi$ be any complex number.

Show that the locus of points for which $\text{Im}\left(\dfrac{z-2}{z}\right) = 1$ is a circle.

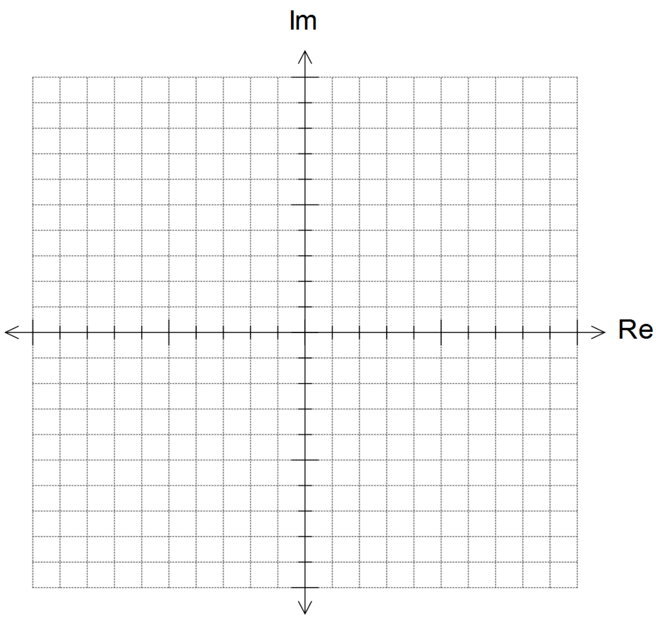
4. **[3 and 3 = 6 marks]**

Sketch the following loci on the complex planes provided.

i) $|z-3| \le |z+3i|$



ii) $\arg\left((1+i)z\right) \ge \dfrac{\pi}{2}$



5. **[5 marks]**

Write down in *cis* (polar) form the solutions to the equation $z^5 = \frac{1}{64}\left(-1+\sqrt{3}\,i\right)$.

6. **[4, 4 = 8 marks]**

(a) When the polynomial $z^2 + (2 - i)z + Ai + B$ is divide by $z + 2i$ the remainder is $2 + 4i$. Determine the values of A and B.

(b) Consider the polynomial $Q(z) = z^4 - 6z^3 + 5z^2 + 22z + 38$. Given that one solution to the equation $Q(z) = 0$ is $z = 4 - \sqrt{3}\, i$, find the other solutions.

7. [2, 2 and 5 = 9 marks]

(a) Given that $z = cis\theta$,

i) prove that $z - \frac{1}{z} = 2i\sin(\theta)$

ii) use de Moivre's Theorem to prove that $z^n + \frac{1}{z^n} = 2\cos(n\theta)$

7 Continued

(b) Use the results from part (a) to show that

$\sin^4\theta - \sin^2\theta = a\cos(4\theta) - b$, giving the values of a and b.

_____End of Test_____